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REVISION NO. \_\_\_\_\_

Project No. E-16-699

DATE 4/12/82

Project Director: Dr. S. G. Lekoudis

School/Lab AE

Sponsor: Office of Naval Research; Arlington, VA

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GTRI/81\*

Title: Turbulent Boundary Layers Development over Compliant Surfaces

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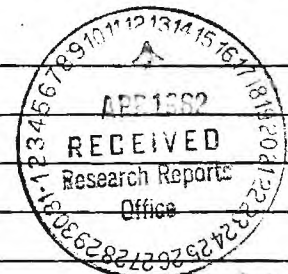
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Date June 16, 1983

Project Title: Turbulent Boundary Layers Development Over Compliant Surfaces

Project No: E-16-699

Project Director: Dr. S. G. Lekoudis

Sponsor: Office of Naval Research

Effective Termination Date: 1/31/83

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SEMI-ANNUAL PROGRESS REPORT  
ON CONTRACT E-16-699 (ONR N00014-82-k-0271)

TURBULENT BOUNDARY LAYERS  
DEVELOPING OVER COMPLIANT SURFACES

Prepared by

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The Office of Naval Research  
Compliant Coating Drag Reduction Program

September 27, 1982

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## 1. Introduction

This progress report covers the period between March 1, 1982 and August 31, 1982. A presentation of this work was made in a Contractor's Review Meeting (Reference 1). The objective of this research program is to develop prediction techniques for high Reynolds number turbulent flows over compliant surfaces (Reference 2). This objective is being pursued by evaluating the wall induced Reynolds stresses using a solution of the linear momentum equations. Thus, before the compliant wall case is examined, as a test of the prediction method, we computed the turbulent boundary layer flow over stationary wavy walls. This is reported in Section 2 of this report. The case of the moving wall is examined in Section 3.

One doctoral student Mr. T. Sengupta is currently involved with the project.

## 2. The Turbulent Boundary Layer Flow Over a Stationary Wavy Wall

Prediction methods for two-dimensional turbulent boundary layers at high Reynolds numbers over stationary wavy walls have been developed using several techniques.

The solution of the linear momentum equations in a body-fitted boundary layer coordinate system is reported in References 3, 4 and 5. However the freestream boundary conditions for the disturbance flow are applied in a non-asymptotic form at certain distances away from the wall.

Moreover the coordinate system used required matching with a cartesian system away from the wall.

The boundary layer equations have been also used to predict such flows . (Article by Cary, Weinstein and Bushnell in Reference 6.) Using this approach necessitates that the pressure is obtained using other equations. One of the choices was to use Miles' theory.

The two-dimensional time-averaged Navier-Stokes equations have also been used (References 1 and 7). The total drag predictions obtained agree well with unpublished data from NASA Langley. The predictions shown for the data of Reference 8 are of about the same quality as the ones shown in Reference 5. The code is capable of handling arbitrary two-dimensional geometries. However it uses periodic boundary conditions and thus, for a surface with multiple waves the parallel flow assumption might lead to inaccuracies, if the waves extend over several boundary layer thicknesses.

The accuracy of the incoming measured boundary layer profile is important in comparing theory with experiment. This statement applies to all predictive techniques.

The linear momentum equations have been solved for turbulent flows over stationary wavy walls. The reason for these calculations is that they serve as a test of the solver for the case of steady turbulent boundary layer flows. The curvilinear system used avoids the problems encountered before. Moreover the freestream boundary conditions are applied in their asymptotic form, appropriate for boundary layer flows.

### 2.1 The Coordinate System

The coordinate system can be used for a swept wavy wall under a three-dimensional turbulent boundary layer. It is constructed from the streamlines and equi-potential lines of inviscid irrotational flow over wavy walls and is shown in Figure 1. It does not have the singularity of the boundary layer coordinate system and because it approaches the cartesian system away from the wall, the correct boundary conditions for the disturbance flow can be applied as appropriate for boundary layer flows. For the data of Reference 8 it was found by trial and error that the region of integration normal to the wall has to extend about 18 displacement thicknesses before the boundary conditions are appropriately applied.

### 2.2 The Governing Equations

In a three-dimensional turbulent boundary layer flow over a wavy surface the velocities  $u$ ,  $v$  and  $w$  (in the directions of  $s$ ,  $n$  and  $q$  respectively) can be expanded as shown in equations (1). There bars denote time-averages and primes denote "random" turbulence fluctuations. Only the real part of the periodic components has any physical meaning, and  $k$  is the wave-number of the wavy wall. After time averaging and linearizing one can obtain a system of equations as shown in equation (2). For the simple case of two-dimensional flow over a two-dimensional wavy wall the system is shown in equations (3). The boundary conditions are that the disturbance flow vanishes on the wavy surface and decays asymptotically away from that surface.

$$u(s, n, q, t) = \bar{U}(n) + u'(s, n, q, t) + \tilde{u}(n) e^{iks} \quad (1a)$$

$$v(s, n, q, t) = \bar{v}(s, n, q, t) + \tilde{v}(n) e^{iks} \quad (1b)$$

$$w(s, n, q, t) = \bar{w}(n) + w'(s, n, q, t) + \tilde{w}(n) e^{iks} \quad (1c)$$

$$p(s, n, q, t) = \bar{P}(s, q) + \tilde{p}(n) e^{iks} \quad (1d)$$

$$\frac{d}{dn} \begin{bmatrix} \tilde{u}(n) \\ \frac{d\tilde{u}(n)}{dn} \\ \tilde{v}(n) \\ \tilde{p}(n) \\ \tilde{w}(n) \\ \frac{d\tilde{w}(n)}{dn} \end{bmatrix} = \begin{bmatrix} \text{depending} \\ \text{on mean} \\ \text{flow} \\ \text{quantities} \end{bmatrix} \begin{bmatrix} \tilde{u}(n) \\ \tilde{v}(n) \\ \tilde{w}(n) \\ \tilde{p}(n) \\ \tilde{w}(n) \\ \frac{d\tilde{w}(n)}{dn} \end{bmatrix} + \begin{bmatrix} \text{Terms depending} \\ \text{on the curvature} \\ \text{and the mean} \\ \text{flow} \end{bmatrix} \quad (2)$$

$$z_1 = \tilde{u}, \quad z_2 = \frac{d\tilde{u}}{dn}, \quad z_3 = \tilde{v}, \quad z_4 = \tilde{p} \quad (3.1)$$

$$\frac{dz_1}{dn} = z_2 \quad (3.2)$$

$$\begin{aligned} \frac{dz_2}{dn} = & (ik \bar{U} R + k^2) z_1 + R \frac{d\tilde{u}}{dn} z_3 + ik R z_4 \\ & + R k e^{-kn} (k \overline{(u'v')})_m + \frac{d}{dn} (\overline{(u'v')})_m - \frac{2}{R} \frac{d^2 \bar{U}}{dn^2} \\ & + R \left( r \frac{d}{dn} (\overline{(u'v')})_m + (\overline{(u'v')})_m \frac{dr}{dn} \right) \end{aligned} \quad (3.3)$$

$$\frac{dz_3}{dn} = -ikz_1 + ik^2 e^{-kn} \bar{U}$$

$$\frac{dz_4}{dn} = -\frac{ik}{R} z_2 - \left(\frac{k^2}{R} + ik \bar{U}\right) z_3 \quad (3.4)$$

$$+ k^2 e^{-kn} \left[ \bar{U}^2 + i(\bar{u}\bar{v})_m - \frac{i}{R} \left( k \bar{U} + \frac{d\bar{u}}{dn} \right) \right] + ikr(\bar{u}\bar{v})_m$$

$$\bar{u}\bar{v} = (\bar{u}\bar{v})_m (1 + r(n) e^{iks}) \quad (3.5)$$

These equations degenerate to the ones appropriate for a cartesian system away from the wall when  $n$  becomes large. Thus the decaying mode of solution can be appropriately applied. Also the Reynolds stresses are composed of their mean value and a periodic component,  $r(n)$ , that needs to be modeled.

In equations (3),  $R$  represents the Reynolds number, and the local freestream velocity and displacement thickness were used for nondimensionalization.

### 2.3 The Numerical Procedure

The mean flow was obtained from a two-dimensional, incompressible, turbulent boundary layer code. The code uses the Keller box finite-difference scheme and the Cebeci-Smith eddy viscosity model. The code has been tested before the initiation of this work for flat plate and airfoil flows.

The stiffness of the system (3) requires the usual Orr-Sommerfeld solution procedures. A well tested method developed over a period of several years by Sandia Laboratories was used (Reference 9). The code uses a Gram-Schmidt orthonormalization procedure. It was found that turbulent mean profiles require greater number of orthonormalizations than laminar profiles. Depending on the accuracy imposed on the Runge-Kutta integrator, about 50 orthonormalizations

were used.

## 2.4 Results

Measurements of turbulent boundary layer flows over wavy walls have been reported in the open literature (References 8 and 10). Reference 8 provides information about the displacement thickness of the incoming boundary layer flow for several heights  $T$  of the trip wire. These were matched with the turbulent boundary layer code and the computed pressure and skin friction distributions are shown in Figures 2 through 11. The  $C_p$  denotes the pressure coefficient,  $C_f$  the local skin friction coefficient,  $C_{fo}$  the flat wall skin friction coefficient and  $Y$  the distance away from the wall at the trough of the wave (Figure 11).

These calculations were done with  $r(n) = 0$ , which means that the Reynolds stresses are those of a flat plate flow. Thus the computed skin friction does not agree well with the measurements, which also have a degree of uncertainty. However the good pressure prediction indicates once more that the influence of the wave is confined mostly in the inner part of the turbulent boundary layer.

Comparison with the data of Reference 10 was also attempted. Data on incoming profiles were not provided. However, the "mean" skin friction over a wave was given using the "mean" measured velocity profile. If the given freestream conditions were used, the boundary layer code predicted lower skin friction than the given values and, as a result, the solution of the linear momentum equations predicted flow separation. When the given value of the mean skin friction was used, the quality of the prediction was comparable to the one for Reference 8.

## 2.5 Work in Progress

The modeling of  $r(n)$  is currently being investigated. The model D of References 3, 4 and 5 is used. However the variation of the disturbance shear in



the inner part of the layer is accounted for, contrary to the previous use of model D. Also, the suggestion of Reference 8 about the relationship of the Reynolds stresses over the wavy wall with those of the flat wall is also being investigated.

### 3. The Moving Wall Case

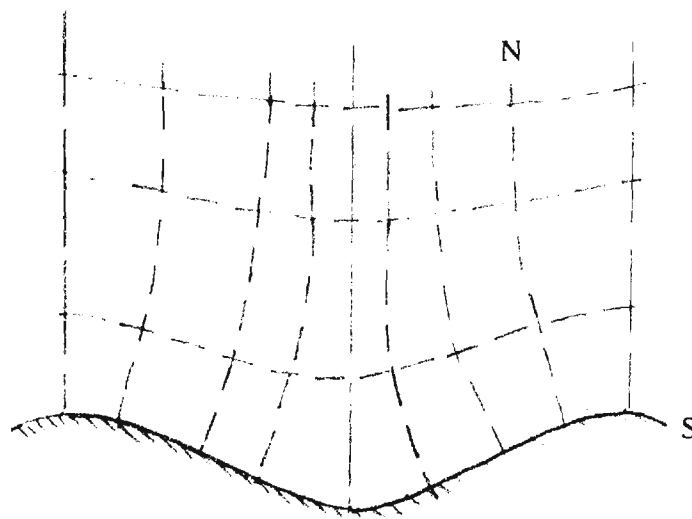
The moving wavy wall case was also investigated, but only in the cartesian coordinate system. This is the first step in the investigation of the compliant walls. Again  $r(n) = 0$  was used, which means that the Reynolds stresses were unaltered from their flat wall values. The variations of the amplitude of the pressure coefficient, the amplitude of the skin friction coefficient and the variation of their phases, with the wall speed, are shown in Figures 12 through 15. The effect of a streamwise pressure gradient was investigated. The conditions correspond to the experiments of Reference 8 for wall I but with 1/10 of the wave amplitude. This is because the cartesian system predicted flow separation with the original amplitude. For the pressure gradient cases the freestream velocity was linearly increased and decreased from its initial value and the local  $\beta = (s/u_e)(du_e/ds)$  at the station where the computed results are shown is -.73, 0, .98 respectively. Because of the cartesian system and the quasi-laminar assumption for the Reynolds stress used, the findings are of qualitative value.

It was found that the magnitude of the skin friction variation decreases slowly with the wall speed. Also the location of the maximum pressure and shear, measured in degrees upstream of the wave crest, showed a very mild dependence on the surface speed. The  $C_{f0}$  symbol denotes the local flat wall value with the pressure gradient. Surface speeds higher than the ones shown were considered unrealistic from an engineering viewpoint, although this is not a restriction of the code.

#### 3.1 Work in Progress

The curvilinear system with model D is used for the moving wall case. Also a marching scheme is being examined for the turbulent boundary layer flows over compliant walls with prescribed wall motion. In this scheme the wall-induced

Reynolds stresses are computed using the local, mean profile and the linear momentum equations. The mean profile is iteratively updated with the new stresses explicitly included in the calculation. Thus, the effect of a compliant wall in the downstream development of a high Reynolds number turbulent boundary layer can be accounted for.



A-A (Not to Scale)

$$S = x + e^{-ky} \sin kx$$

$$n = y - e^{-ky} \cos kx$$

$$q = z$$

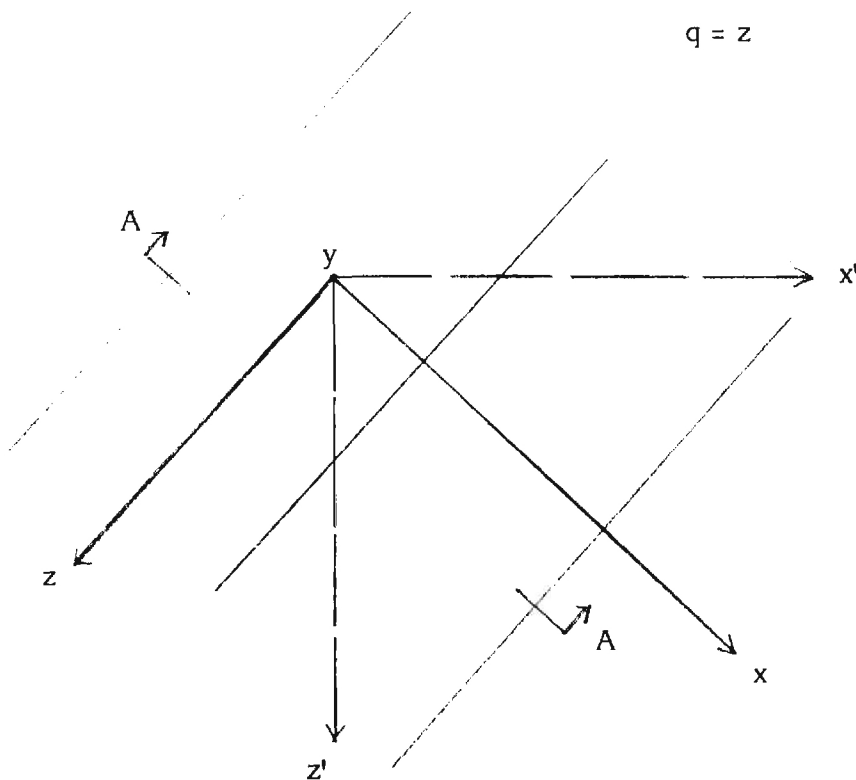


Figure 1. The Coordinate System

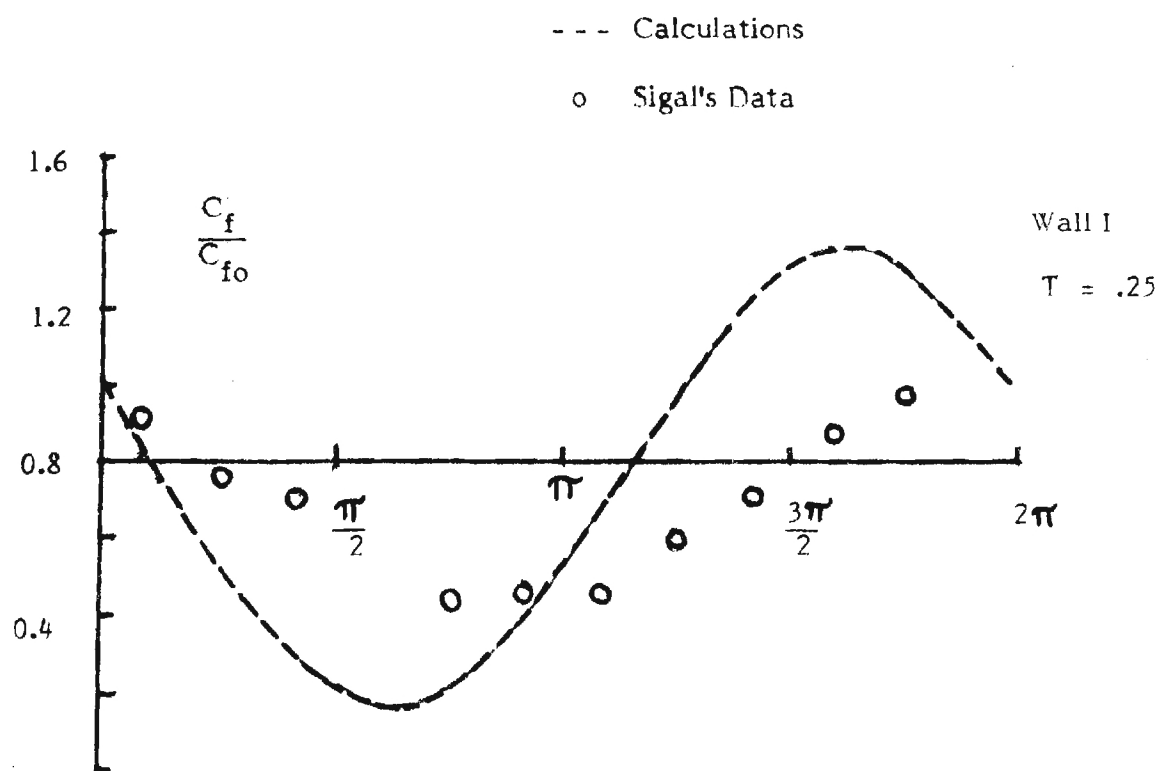


Figure 2

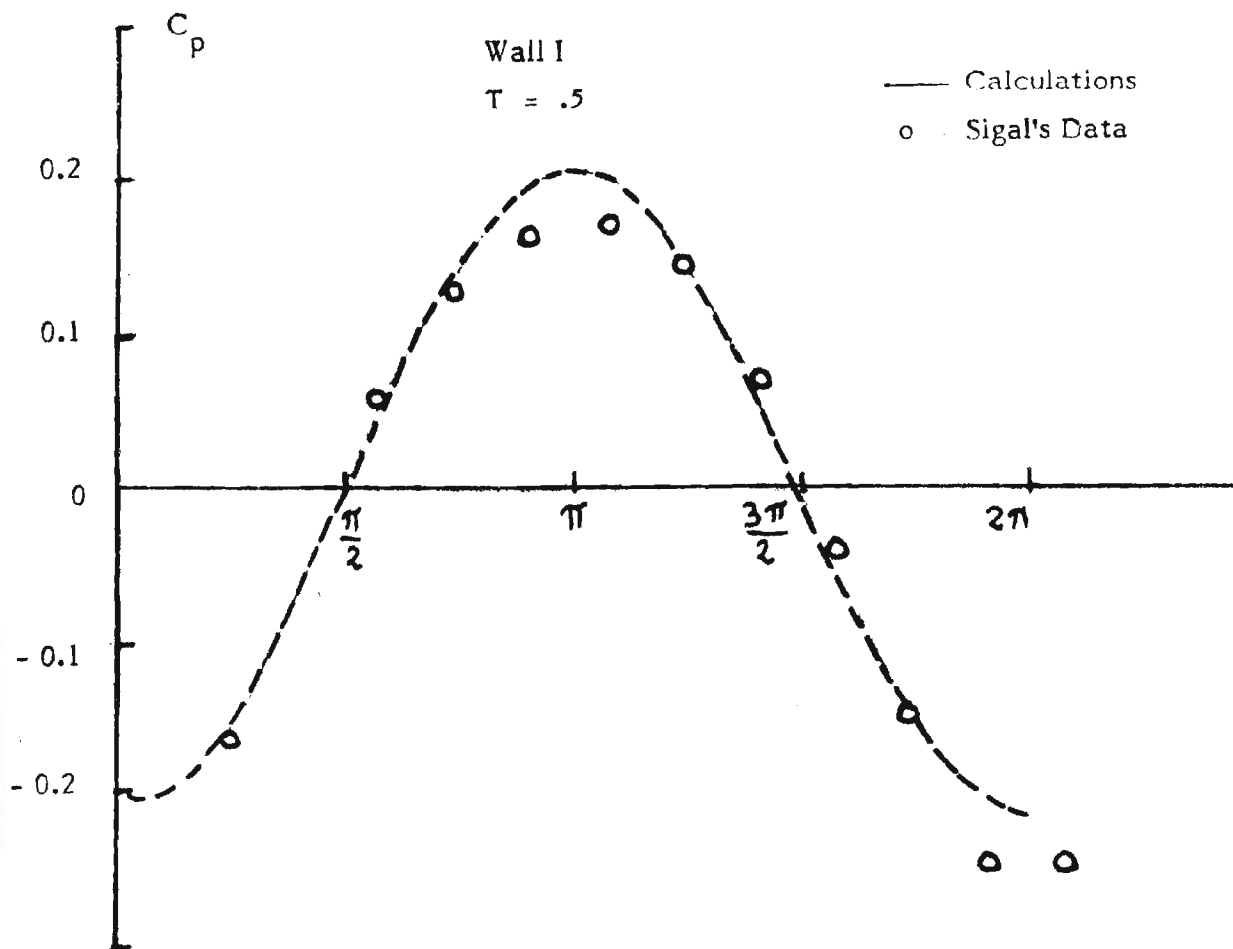


Figure 3



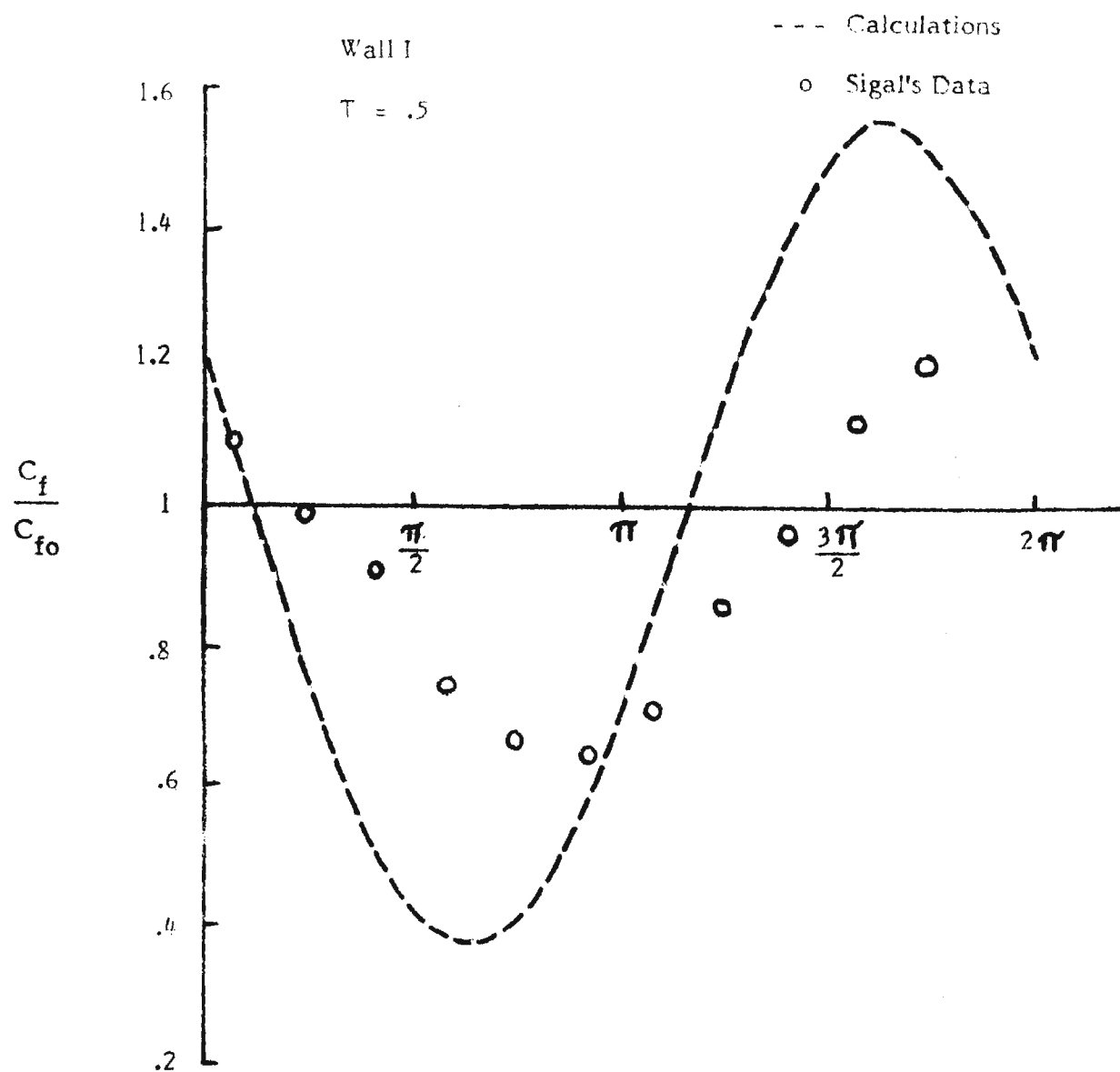


Figure 4

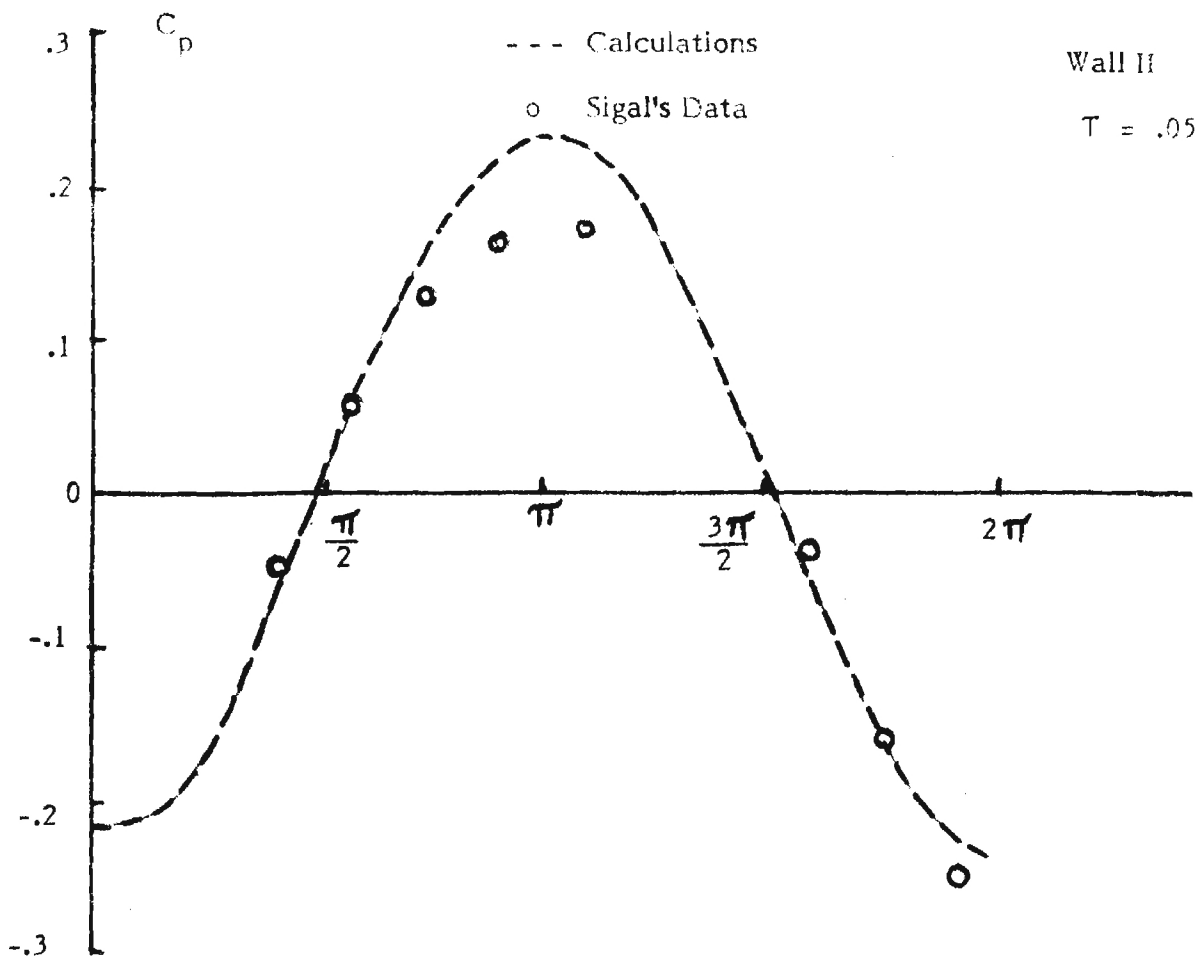


Figure 5

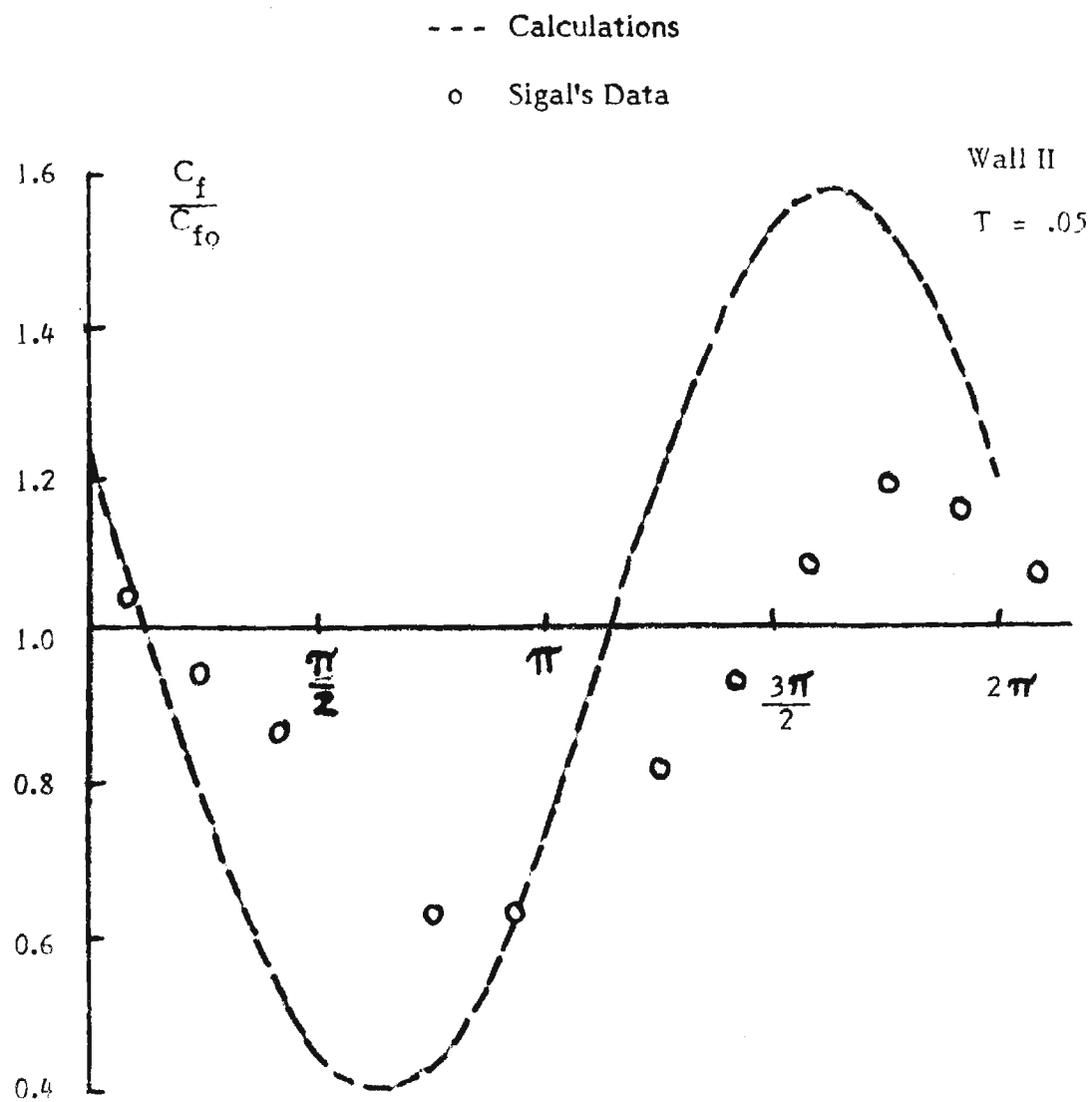


Figure 6

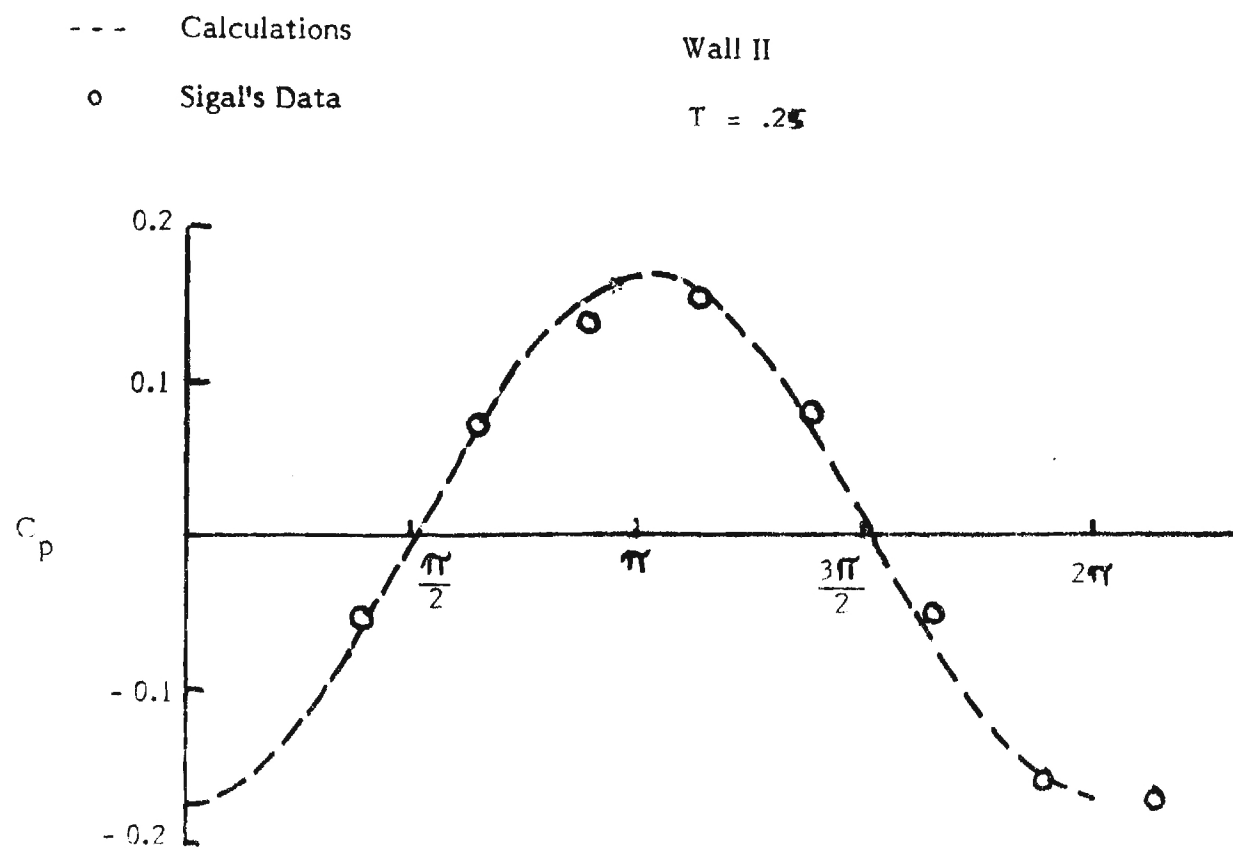


Figure 7

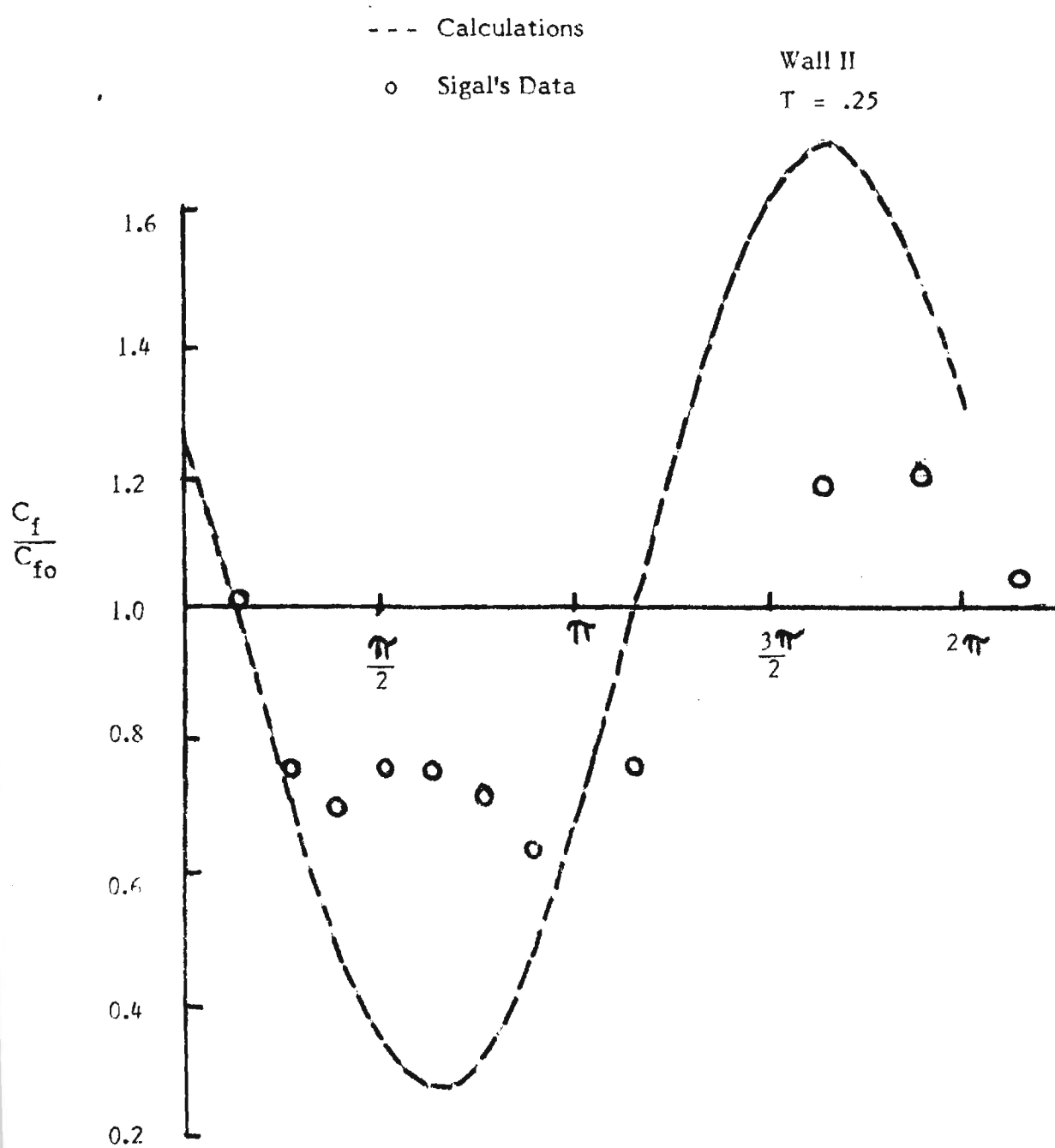


Figure 8

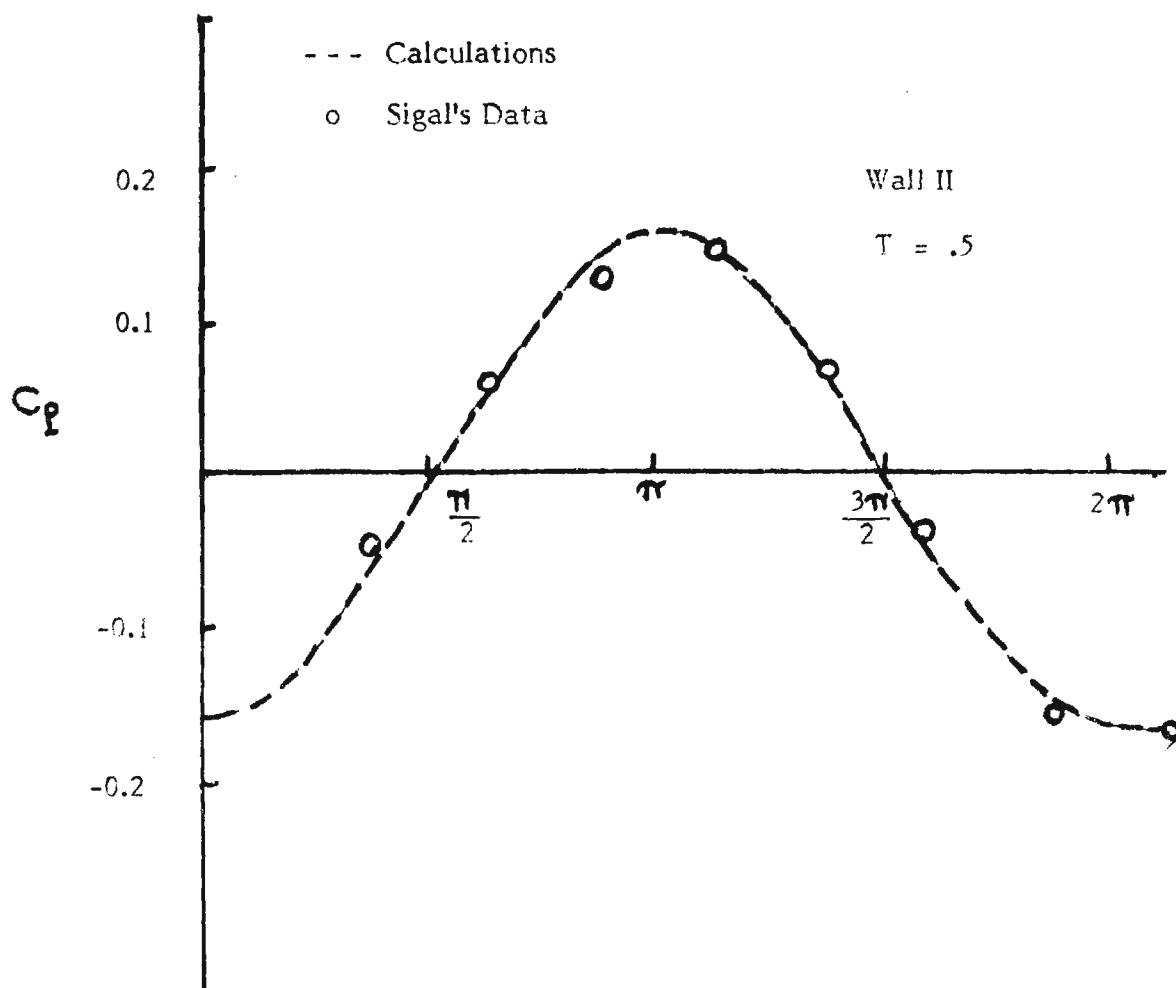


Figure 9



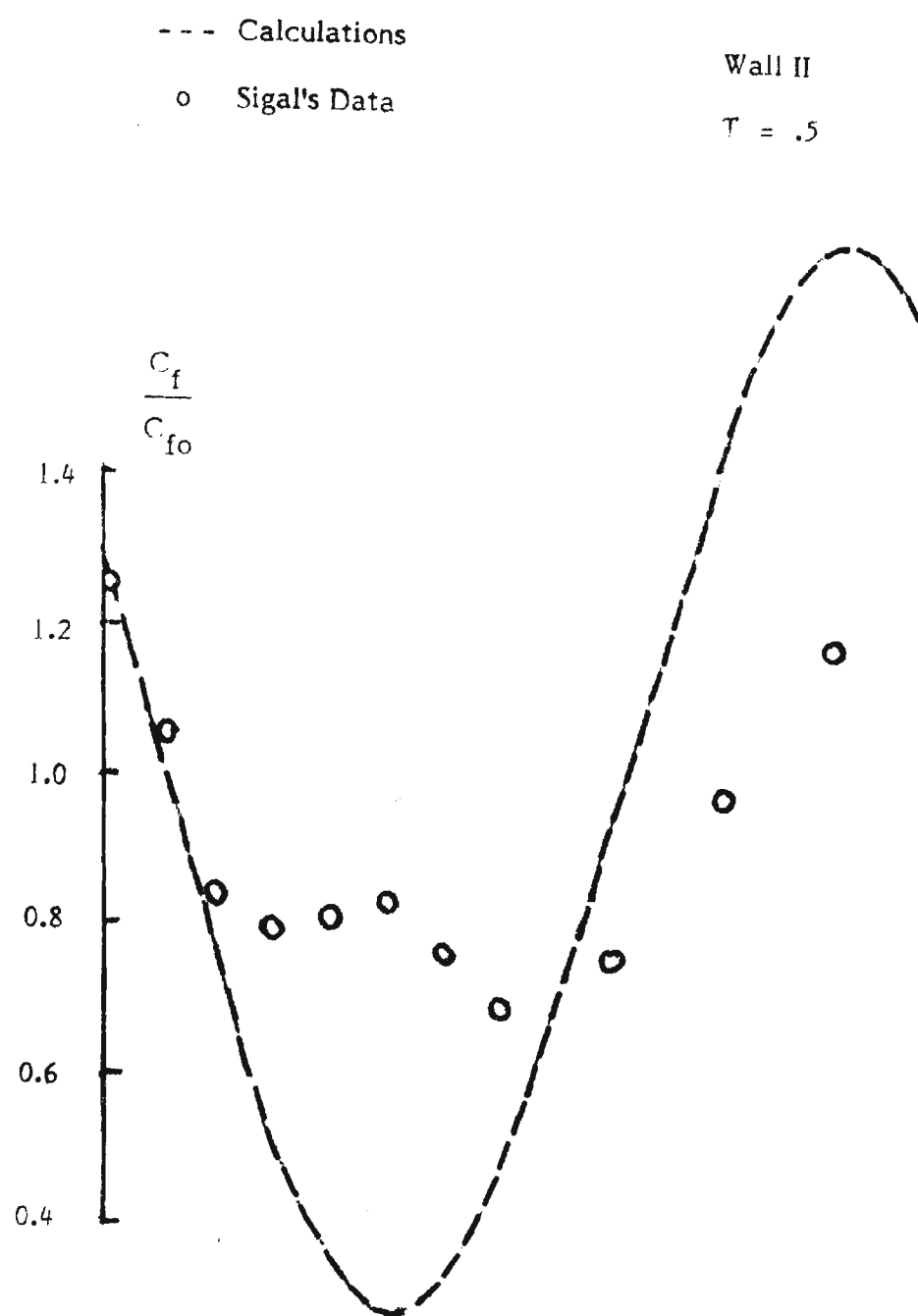


Figure 10

Wall II

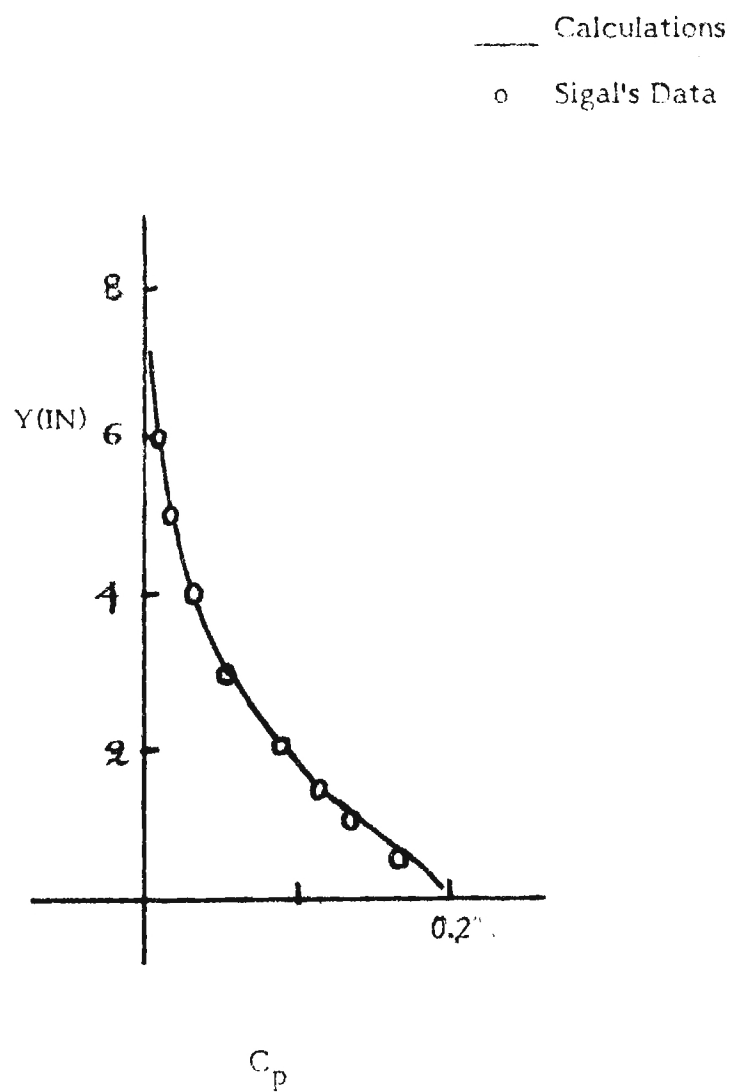
 $T = .5$ 

Figure 11

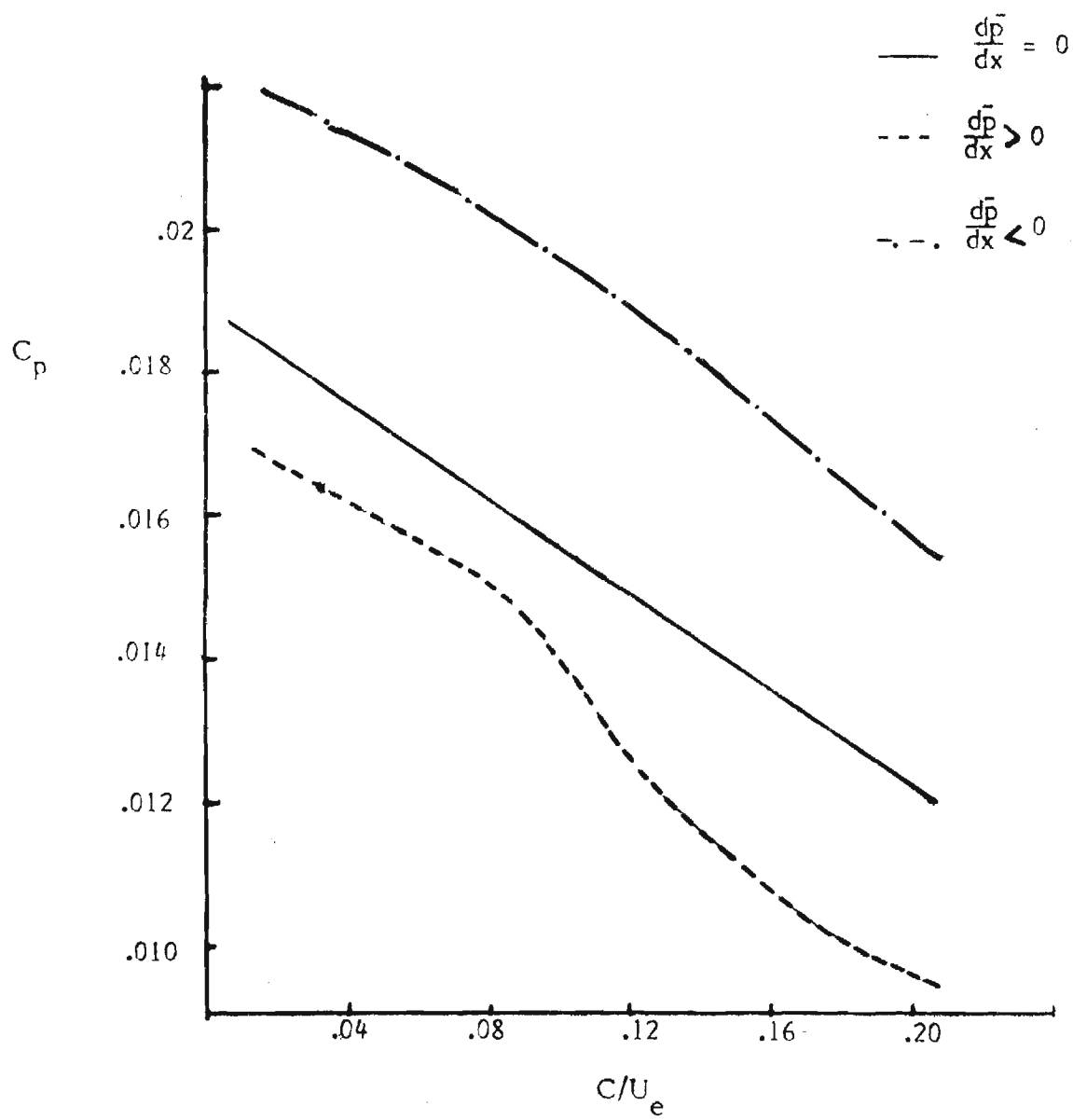


Figure 12

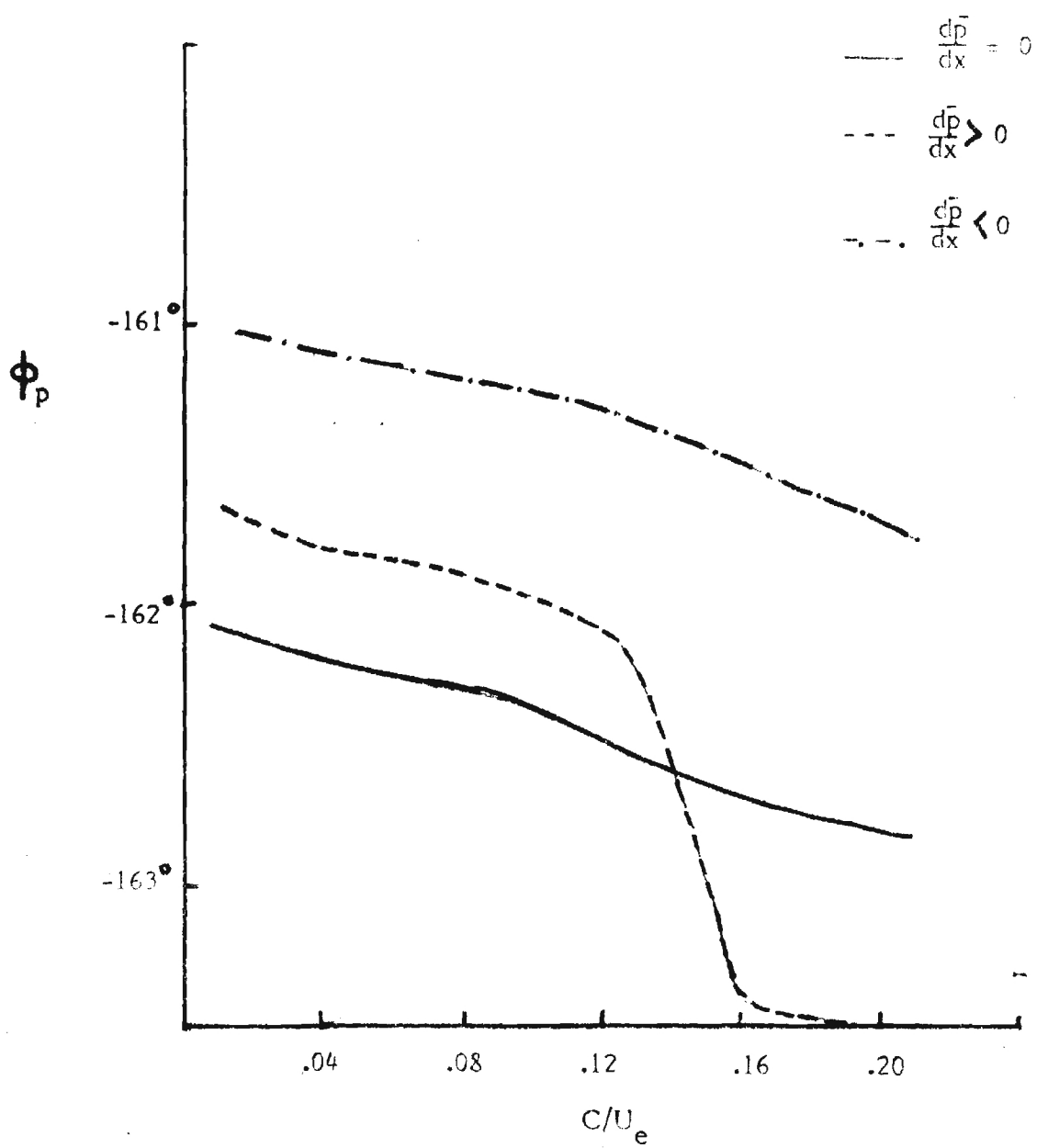


Figure 13

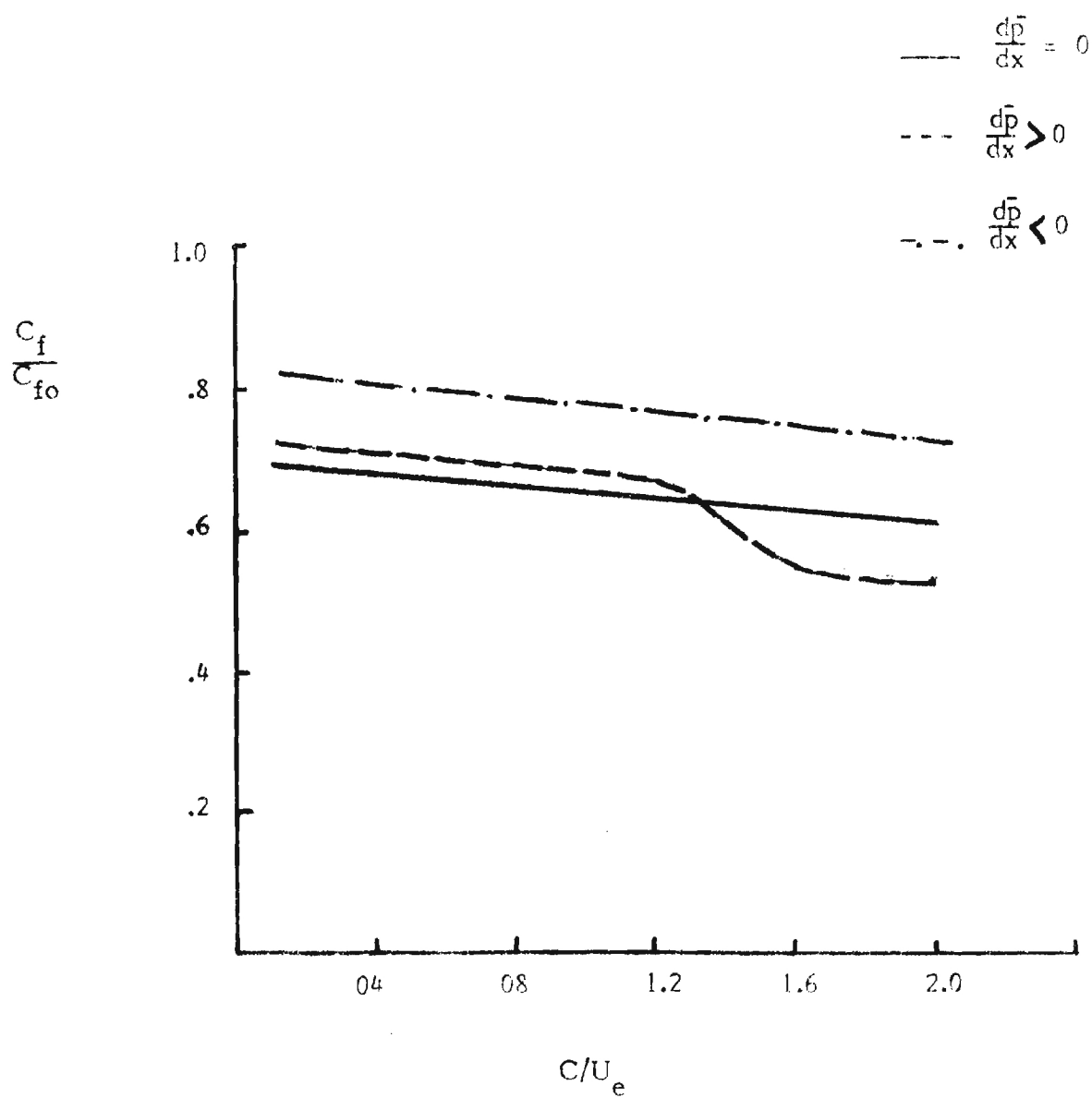


Figure 14

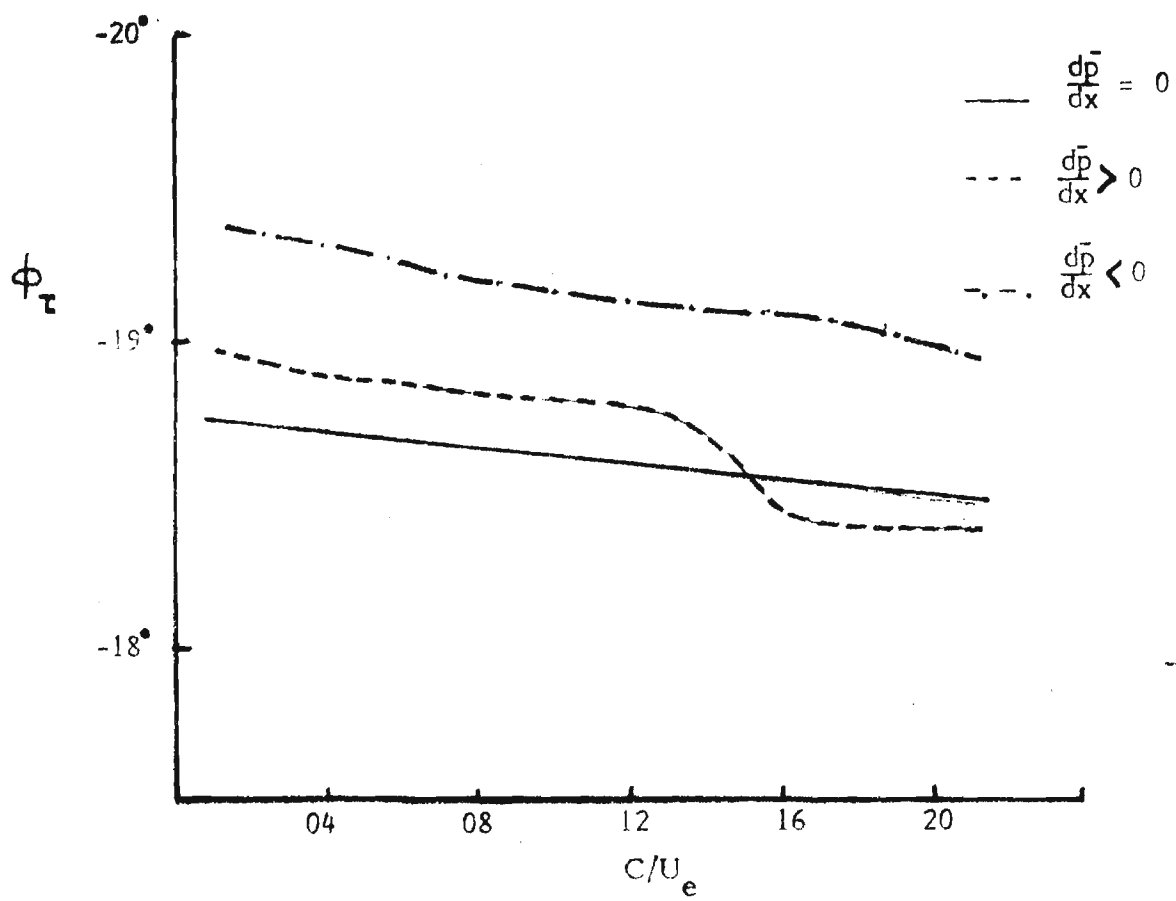


Figure 15



#### 4. References

1. Drag Reduction Symposium, National Academy of Sciences, Washington, D.C., September 13-17, 1982. Sponsored by ONR, NASA, AFOSR and NSSC.
2. "Turbulent Boundary Layers Developing Over Compliant Surfaces" Research Proposal by S. G. Lekoudis, submitted by the Georgia Tech Research Institute to the Office of Naval Research in December 1980.
3. "Influence of the Amplitude of a Solid Wavy Wall on a Turbulent Flow, Part 1, Non-Separated Flows" by D.P. Zilker, G. W. Cook and T.J. Hanratty, J. Fluid Mechanics, Vol. 82, Part 1, pp. 29-51, 1977.
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7. "Numerical Studies of Laminar and Turbulent Drag Reduction" by R. Balasubramanian and S. Orszag, NASA Contractors Report 3498, December 1981.
8. "An Experimental Investigation of the Turbulent Boundary Layer Over a Wavy Wall," by A. Sigal, Ph.D. Thesis, California Institute of Technology, 1971.
9. "Computational Solution of Linear Two Point Boundary Value Problems Via Orthonormalization," by M.R. Scott and H.A. Watts, SIAM Journal Numerical Analysis, Vol. 14, No. 1, 1977.
10. "The Turbulent Boundary Layer Over a Wall with Progressive Surface Waves," by J. Kendall, J. Fluid Mechanics, Vol. 41, Part 2, pp. 259-281, 1970.

**FINAL REPORT**

**TURBULENT BOUNDARY LAYERS  
DEVELOPING OVER COMPLIANT SURFACES**

**By**

**S. G. Lekoudis and T. Sengupta**

**Prepared for**

**THE OFFICE OF NAVAL RESEARCH  
COMPLIANT COATING DRAG REDUCTION PROGRAM**

**Under**

**Contract ONR N00014-82-K-0271  
(Georgia Tech Research Institute E-16-699)**

**May 1983**

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**A UNIT OF THE UNIVERSITY SYSTEM OF GEORGIA**  
**SCHOOL OF AEROSPACE ENGINEERING**  
**ATLANTA, GEORGIA 30332**

**1983**



Final Report  
on Contract ONR N00014-82-K-0271  
(Georgia Tech Research Institute E-16-699)

TURBULENT BOUNDARY LAYERS  
DEVELOPING OVER COMPLIANT SURFACES

By

S. G. Lekoudis and T. Sengupta  
School of Aerospace Engineering  
Georgia Institute of Technology

For

The Office of Naval Research  
Compliant Coating Drag Reduction Program

May 6, 1983

## SUMMARY

This report summarizes work done under the ONR Contract No. N00014-82-K-0271 to Georgia Tech, between March 1, 1982 and March 1, 1983. The objective of this research program was to develop prediction techniques for high Reynolds number turbulent flows over compliant surfaces. This objective was pursued by evaluating the wall induced Reynolds stresses using solutions of the linear momentum equations.

One graduate student, Mr. Tapan Sengupta was the research assistant in this program. He is currently a Ph.D. candidate at the School of Aerospace Engineering at Georgia Tech.

## 1. INTRODUCTION

The problem of reducing drag due to skin friction remains of interest. This is the case because of the significant benefits that would result from an application of a drag reducing scheme on airplanes, ships or underwater vehicles. One of the techniques that have been proposed for such a scheme is wall compliance. Wall compliance could, in principle, work in two ways: either it could delay transition, or it could modify the inner part of a turbulent boundary layer so that reduced skin friction would result.

The Office of Naval Research supports an ongoing program in compliant surfaces for drag reduction. The program consists of analytical and experimental studies with the goal of inventing a working system (Reference 1). The prime candidate for such a system is the turbulent boundary layer developing on a surface with desirable properties. Therefore, the center of attention in this research is the interaction of the turbulent boundary layer with a compliant coating.

Any prediction method that attempts to compute high Reynolds number flow over compliant surfaces aims at predicting unsteady turbulent flow. Therefore some credibility of the method must be established by using it to predict steady turbulent flows over rigid wavy surfaces. There are measured data for such flows (References 2, 3, 4, 5). Such flows do not show any beneficial drag reduction. The reason is that although the average skin friction is lower than that for the equivalent flat plate flow, the phase shift of the pressure results in a net drag increase.

The developed prediction technique and results for both stationary and moving wavy surfaces are presented and discussed in the next sections of this report.

## 2. THE TWO-DIMENSIONAL TURBULENT BOUNDARY LAYER OVER A STATIONARY WAVY WALL IN INCOMPRESSIBLE FLOW

The coordinate system used (Figure 1) is boundary-conforming. This is very important because the linearized wall boundary condition (transfer at the mean interface) presents serious errors for wave amplitudes that are large compared to the turbulent sublayer thickness, if a cartesian system is used (Reference 6). This error was numerically verified by working with a cartesian system and predicting turbulent separation for the unseparated flow of Reference 5. The coordinate system is frozen in space, if one moves in the positive x-direction with the wave velocity  $c$  (zero for the rigid wall case). Contrary to the coordinate systems used in References 6 and 7, the present system has no singularities away from the wall and approaches a cartesian system far from the wall. The normalizing parameters are the local boundary layer displacement thickness  $\delta^*$  and the local freestream velocity  $U_e^*$ . The nondimensional wave amplitude  $\epsilon$  is supposed to be small for the analysis to be valid. For the data of Reference 5 it is more than 1, but this did not seem to invalidate the calculations. It was found, by trial and error, that for the same data the integration normal to the wall has to extend about  $18\delta^*$  before the freestream boundary conditions are appropriately applied.

The velocity (Figure 1) components  $u, v$  in the  $S, N$ , directions respectively are decomposed into three parts: a mean time-independent part, a random part and a periodic in space and time part. For example, the  $u$ -velocity component is written as

$$u = \bar{U}(N) + u'(s, N, z, t) + \epsilon \left[ \bar{u}(N) e^{ik(s-ct)} + \bar{u}^*(N) e^{-ik(s-ct)} \right] \quad (1)$$

In Equation (1) stars denote complex conjugates. Time-averaging the Navier-Stokes equations for incompressible flow results into stresses due to the



random component and to the periodic component of the velocity (References 9, 10). The last stresses are termed wave-induced stresses. Taking a phase average of the Navier-Stokes equations, subtracting the time-averaged equations, neglecting terms of order  $\epsilon^2$  and assuming that the difference between the time-averaged stresses due to the random velocity component and the phase-averaged stresses due to the same velocity component is small, we obtain an Orr-Sommerfeld system for the periodic part of the flow.

The Orr-Sommerfeld system is nonhomogeneous because of the terms due to curvature, with homogeneous boundary conditions (References 8, 9, 10). The numerical difficulties associated with such systems are taken care of with the use of standard methods (Reference 11). It was found that, in general, the Orr-Sommerfeld problem for a turbulent mean profile is stiffer than the same problem for a laminar profile. The reason is not the Reynolds number but the profile shape. The mean turbulent flow is solved using a finite-difference technique, the Keller-box, for two-dimensional incompressible flow with arbitrary pressure gradients.

The computed pressure and skin friction variations were compared with measured data. A point has to be made about the stresses from the random component of the velocity. If these stresses remain unexpanded in the perturbation scheme, the agreement with measured data is not as good, especially for the skin friction distribution (References 8, 9). The model used in References 2, 3 and 6 proved adequate. Navier-Stokes simulations of these steady turbulent flowfields (References 4 and 12) indicate that the algebraic eddy-viscosity is adequate. This model is used for the mean flow calculations.

Comparisons for pressure and skin friction distributions are shown in Figures 2, 3, and 4. The measurements are from Reference 5 for the Figures 2 and 3 and from Reference 4 for Figure 4. No detailed skin-friction measurements are

available for the flows described in Reference 4. The agreement between calculations and measurements is good. It should be emphasized that good agreement with measured pressures is not very hard to achieve, even simpler theories show good qualitative agreement. The skin friction is a much more difficult quantity to predict for such flows.

The linear theory can provide estimates of the pressure drag, but not of the skin friction drag. This is because to  $O(\epsilon)$  the periodic variation of the skin friction produces no net effect on the drag. However it is experimentally established that the mean skin friction of these flows is lower than that of the flat plate (Reference 4). This is the reason such flows were examined. To predict the average skin friction, we used a nonlinear theory (Reference 9). The boundary layer equations used to obtain the mean flow  $O(N)$ , contain wave-induced stresses. These stresses are:

$$-\frac{\epsilon^2}{4}(\tilde{u} \tilde{v}^* + \tilde{u}^* \tilde{v}) \quad (2)$$

and they are functions of  $N$  only. Stars in Equation (2) denote complex conjugates. The wave-induced stresses described in Equation (2) represent essentially a streaming effect. The boundary layer equations were solved including these stresses in an iterative fashion. The iterations were between the meanflow solution and the solution of the Orr-Sommerfeld system (Reference 9). No more than 5 iterations are needed to obtain a converged solution. The following observations can be made from this solution.

The mean flow is slightly modified only close to the wall with a resulting reduction in the skin friction. Figure 5 shows the skin friction distribution for the experimental setup of Reference 4. The calculations predict that the skin friction is lower than that of the flat plate, in agreement with experimental observations. The iterative scheme has a very small effect on the amplitude and the phase shift



of the pressure and the shear on the wavy wall. The predicted drag was compared with the data from NASA-Langley (Reference 4). For the wave with  $h/\lambda = .015$  (Reference 4), the ratio of the pressure drag over the flat plate drag was computed to be 0.234 and the same ratio for the skin friction drag was 0.985. The agreement is comparable with the one obtained from solutions of the full Navier-Stokes and in excellent agreement with the measurements (References 4, 9). The code used to generate these predictions takes about one hour of CDC 6600 time for the 35 sinusoidal waves of the experimental setup of Reference 4.

### 3. THE TWO-DIMENSIONAL TURBULENT BOUNDARY LAYER OVER A MOVING WAVY WALL IN INCOMPRESSIBLE FLOW

For simplicity, a cartesian system was used to solve the Orr-Sommerfeld system for the case of a wall translating in the flow direction with nondimensional velocity  $c$ . Details from this calculation are given in Reference 8. Because of the coordinate system, the wavy wall of Reference 5 was reduced to  $1/10$  of its amplitude, and then used in the moving wall calculations. The Orr-Sommerfeld system predicted flow separation for the original wave by calculating a periodic skin friction with amplitude larger than the mean skin friction. Very little change in the phase of the shear and the pressure variations resulted from the wall motion. Therefore the linear theory indicates that the only favorable effect from a moving wall is the reduction of the pressure drag because of the reduction of the amplitude of the oscillating pressure (Reference 8). The calculations were repeated for a boundary layer with a pressure gradient. Again no significant changes from the stationary wall case of the phase shift of the pressure and the shear variations were found (Reference 8).

Solutions for the nonlinear problem were also computed. The skin friction distribution, for the experimental setup of Reference 4, for nondimensional wall velocities, of 0.1 and 0.2 are shown in Figure 5. The wavy wall skin friction is slightly lower than that for the flat wall that moves with the same velocity, as expected. The pressure coefficients are shown in Figure 6.

#### 4. THE COMPLIANT WALL PROBLEM IN TWO AND THREE DIMENSIONS

The previous two sections in this report deal with the case of stationary wavy walls, or walls that translate with uniform velocity. In this section the use of the coordinate system for the calculation of compliant wall motion is described.

For the two-dimensional problem, the absolute velocities  $u, v$  in an inertial cartesian  $(x, y)$  coordinate system, on the surface of the wave,  $N = 0$ , are (Reference 9):

$$u = \bar{U} + u' + c + \epsilon e^{ik(s-ct)} [\bar{u} - k(\bar{U} + u') - ikv'] \quad (3)$$

$$v = v' + \epsilon e^{ik(s-ct)} [\bar{v} - kv' + ik(\bar{U} + u')] \quad (4)$$

where the velocities in the RHS of these equations are all relative to the curvilinear  $S, N, Z$  coordinate system. For the case of a translating wall,  $u = c$  and  $v = 0$ , as it should be, because  $\bar{U} = \bar{u} = \bar{v} = u' = v' = 0$  on the wavy surface.

For a compliant surface  $\bar{U} = -c, u' = v' = 0$ , and therefore

$$\bar{u} = \frac{u}{\epsilon e^{ik(s-ct)}} - kc \quad (5)$$

$$\bar{v} = \frac{v}{\epsilon e^{ik(s-ct)}} + ikc \quad (6)$$

For a compliant wall that admits traveling wave solutions  $u$  and  $v$  are multiples of  $e^{ik(s-ct)}$  with coefficients that are related with the wall compliance. Since we are not solving the eigenvalue problem (i.e., material properties of the wall are neglected) we can prescribe  $u$  and  $v$  at will. In this case the Orr-Sommerfeld system has inhomogeneous boundary conditions as well. Calculations for such a system will be reported in Reference 9.

The three-dimensional problem can be more complicated than it appears. The reason is that the direction of the wave motion and the direction of the phase

of the wavy surface do not have to coincide (Figure 7). A coordinate system that moves with the wave and has all the desirable properties of the one used in the two-dimensional problem is constructed in Figure 7. Then the velocity components in the  $S, N, Z'$  curvilinear system can be expanded as follows:

$$u = \bar{U}(N) + u' + \epsilon \left[ \tilde{u} e^{ik(s - c_1 t)} + \tilde{u}^* e^{-ik(s - c_1 t)} \right] \quad (7)$$

$$v = v' + \epsilon \left[ \tilde{v} e^{ik(s - c_1 t)} + \tilde{v}^* e^{-ik(s - c_1 t)} \right] \quad (8)$$

$$w = \bar{W}(N) + w' + \epsilon \left[ \tilde{w} e^{ik(s - c_1 t)} + \tilde{w}^* e^{-ik(s - c_1 t)} \right] \quad (9)$$

where  $\bar{U}$  and  $\bar{W}$  are the projections of the two-dimensional mean velocity profile on the  $xy$  and  $yz$  axis, respectively. The Navier-Stokes equations and the Equations (7), (8) and (9) give another nonhomogeneous Orr-Sommerfeld system, because of the terms due to curvature. For the case of a wavy wall, translating with velocity  $c$  in the  $x$ -direction ( $\theta = 90^\circ$ ) the boundary conditions are homogeneous. The nonlinear problem can be solved by finding the wave-induced stresses in the plane of the mean flow, which is not specified in Figure 7 and is also arbitrary. This can be done by velocity decomposition once the solution from the Orr-Sommerfeld system is known.

For the case of compliant walls, equations analogous to (6) and (7) can be derived. The equations and calculations of such flows will be reported by the authors of this report.

## 5. CONCLUDING REMARKS

Two-dimensional turbulent boundary layers, in incompressible flow, over wavy surfaces have been investigated. The solution of the linear momentum equations, reduced to a nonhomogenous Orr-Sommerfeld system, have been obtained. They have been used to model the wave-induced stresses in the time-averaged boundary layer equations. The calculations predict a small reduction of the mean skin friction, in agreement with experimental observations, for the case of stationary wavy walls. This reduction persists when the wavy wall is translating downstream with uniform velocity, while the pressure drag is decreased.

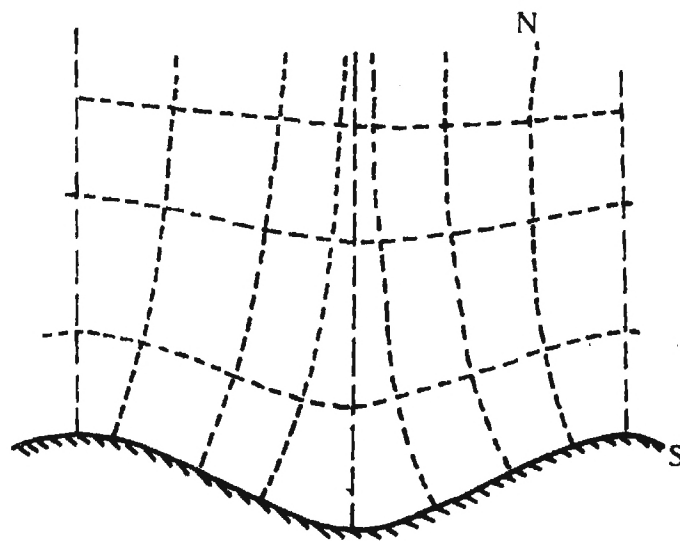
## 6. PUBLICATIONS AND PRESENTATIONS

Based on work supported from this contract, the following publications and presentations resulted:

1. "Calculation of Turbulent Boundary Layers Developing Over Compliant Surfaces," by T. Sengupta and S. Lekoudis, presentation at the Drag Reduction Symposium, in the National Academy of Sciences (Washington, D.C., September 13-17, 1982), sponsored by ONR, NASA, AFOSR and NNSC.
2. "Calculation of Incompressible Turbulent Boundary Layers Over Moving Wavy Surfaces," by T. Sengupta and S. Lekoudis, AIAA Paper 83-1670 (16th Fluid and Plasma Dynamics Conference, July 12-14, 1983).

## 7. REFERENCES

1. Abstracts of the Drag Reduction Symposium held at the National Academy of Sciences in Washington, D.C. on September 13-17, 1982 and sponsored by ONR, NASA, AFOSR, and NSSC.
2. "Influence of the Amplitude of a Solid Wavy Wall on a Turbulent Flow, Part 1, Non-Separated Flows," by D. P. Zilker, G.W. Cook and T.J. Hanratty, J. Fluid Mechanics, Vol. 82, Part 1, pp. 29-51, 1977.
3. "Influence of the Amplitude of a Solid Wavy Wall on a Turbulent Flow, Part 2, Separated Flows," by D.P. Zilker and T.J. Hanratty, J. Fluid Mechanics, Vol. 90, Part 2, pp. 257-271, 1979.
4. "Turbulent Drag Characteristics of Small Amplitude Rigid Surface Waves," by Lin, J.C., Walsh, M.J., Watson, R.D. and Balasubramaniam, R., AIAA Paper 83-0228.
5. "An Experimental Investigation of the Turbulent Boundary Layer Over a Wavy Wall," by A. Sigal, Ph.D. Thesis, California Institute of Technology, 1971.
6. "A Comparison of Linear Theory with Measurements of the Variation of Shear Stress Along a Solid Wave," by C.B. Thorsnees, P.E. Morrisroe and T. J. Hanratty, Chemical Engineering Science, Vol. 33, pp. 579-592, 1978.
7. "Viscous Flow Drag Reduction," Progress in Astronautics and Aeronautics, (Paper by Cary, A.M., Weinstein, L.M. and Bushnell, D.M.), Vol. 72, 1980.
8. "Turbulent Boundary Layers Developing Over Compliant Surfaces," by S. G. Lekoudis and T. Sengupta, Progress Report on Contract N00014-82-0271 submitted to ONR in September 1982.
9. "Turbulent Boundary Layers Over Moving Wavy Surfaces," by T. Sengupta and S. Lekoudis, AIAA Paper 83-1670, 16th Fluids and Plasma Dynamics Conference, Danvers, Mass.
10. "Turbulent Boundary Layers Developing Over Compliant Surfaces," proposal submitted by the Georgia Institute of Technology to ONR in December 1980.
11. "Computational Solution of Linear Two Point Boundary Value Problems Via Orthonormalization," by M.R. Scott and H.A. Watts, SIAM Journal Numerical Analysis, Vol. 14, No. 1, 1977.
12. "Computation of Turbulent Flow Over a Moving Wavy Surface," by John McLean, Bulletin of the American Physical Society, Vol. 27, No. 9, 1982.



$$S = (x - ct) + \epsilon e^{-ky} \sin k(x - ct)$$

$$n = y - \epsilon e^{-ky} \cos k(x - ct)$$

Figure 1. The Coordinate System

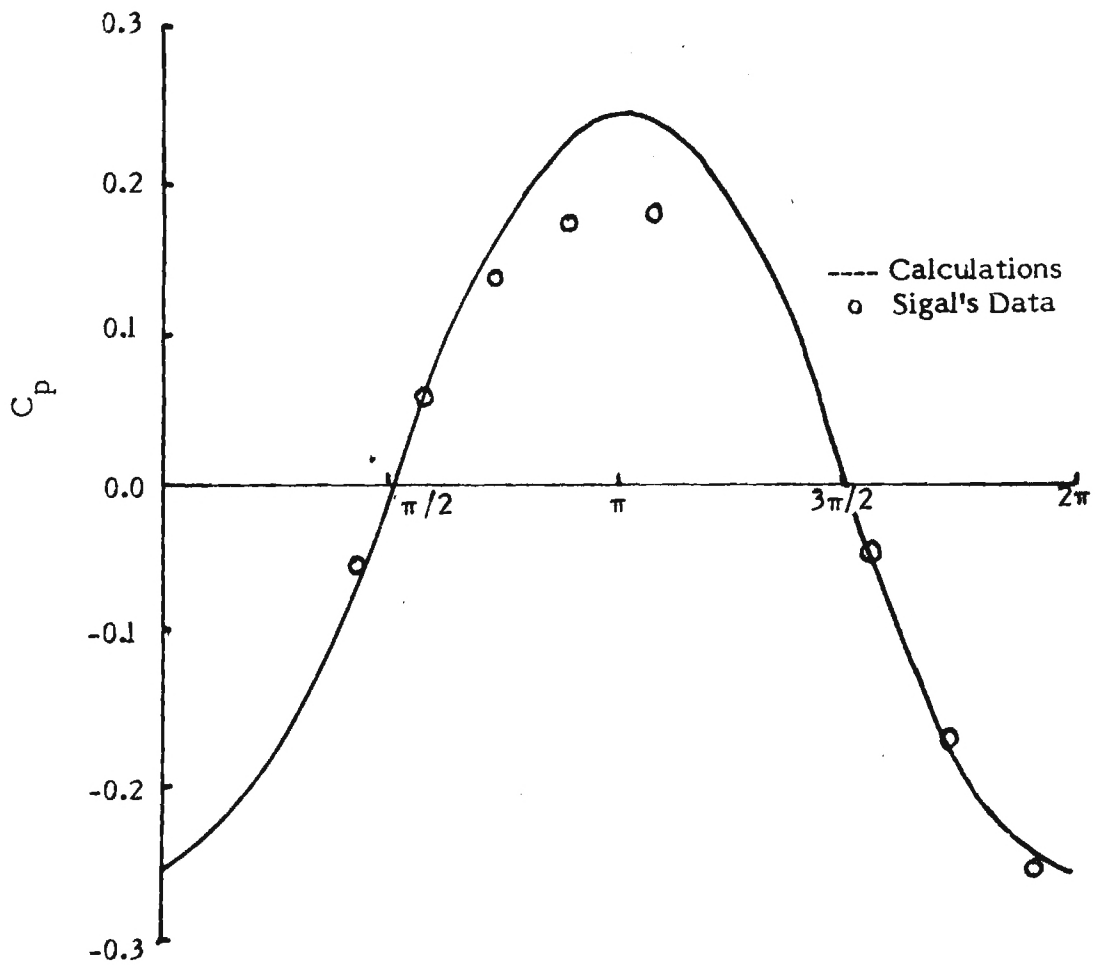


Figure 2. Comparison between calculations and measurements



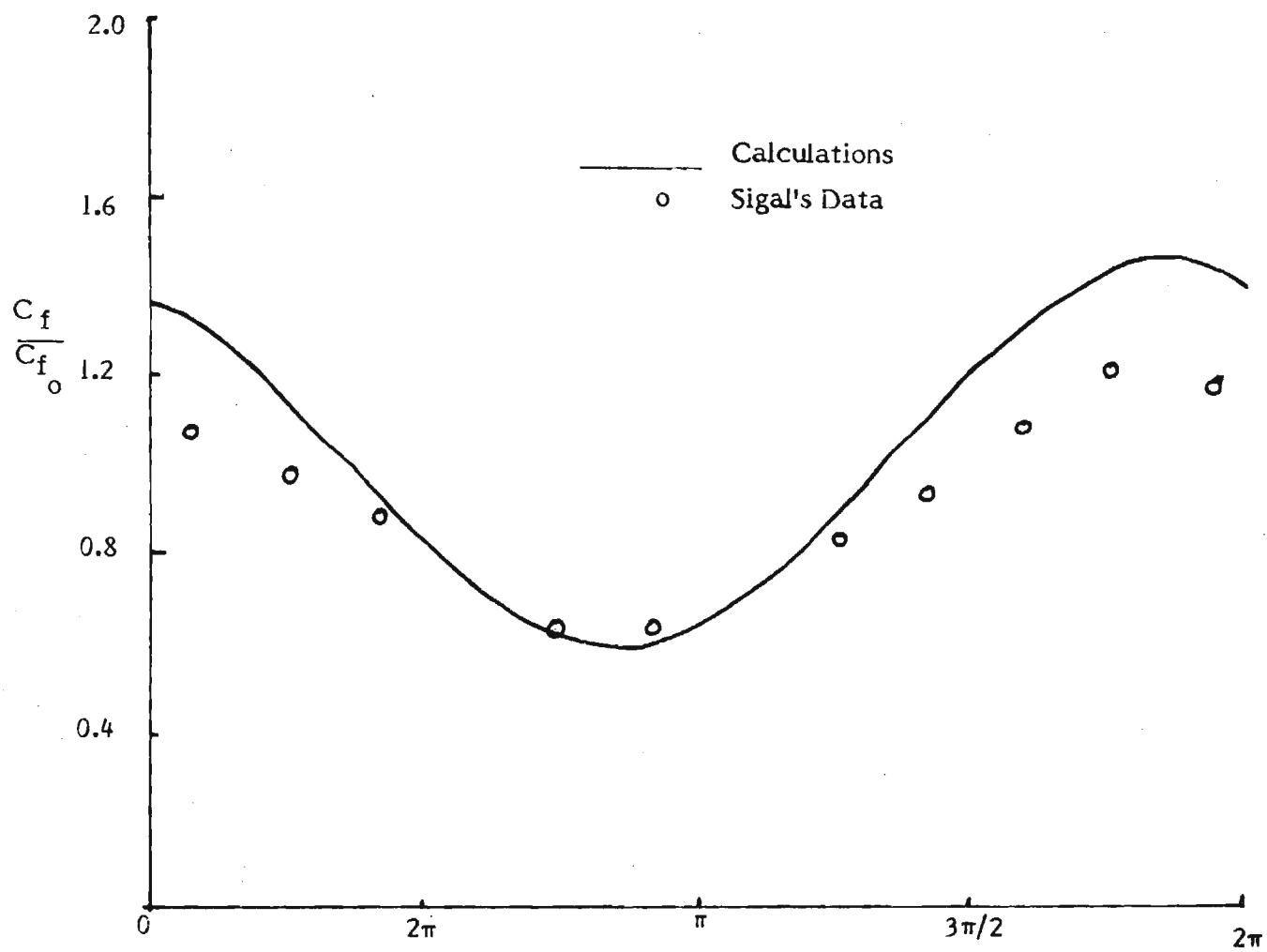


Figure 3. Comparison between calculations and measurements

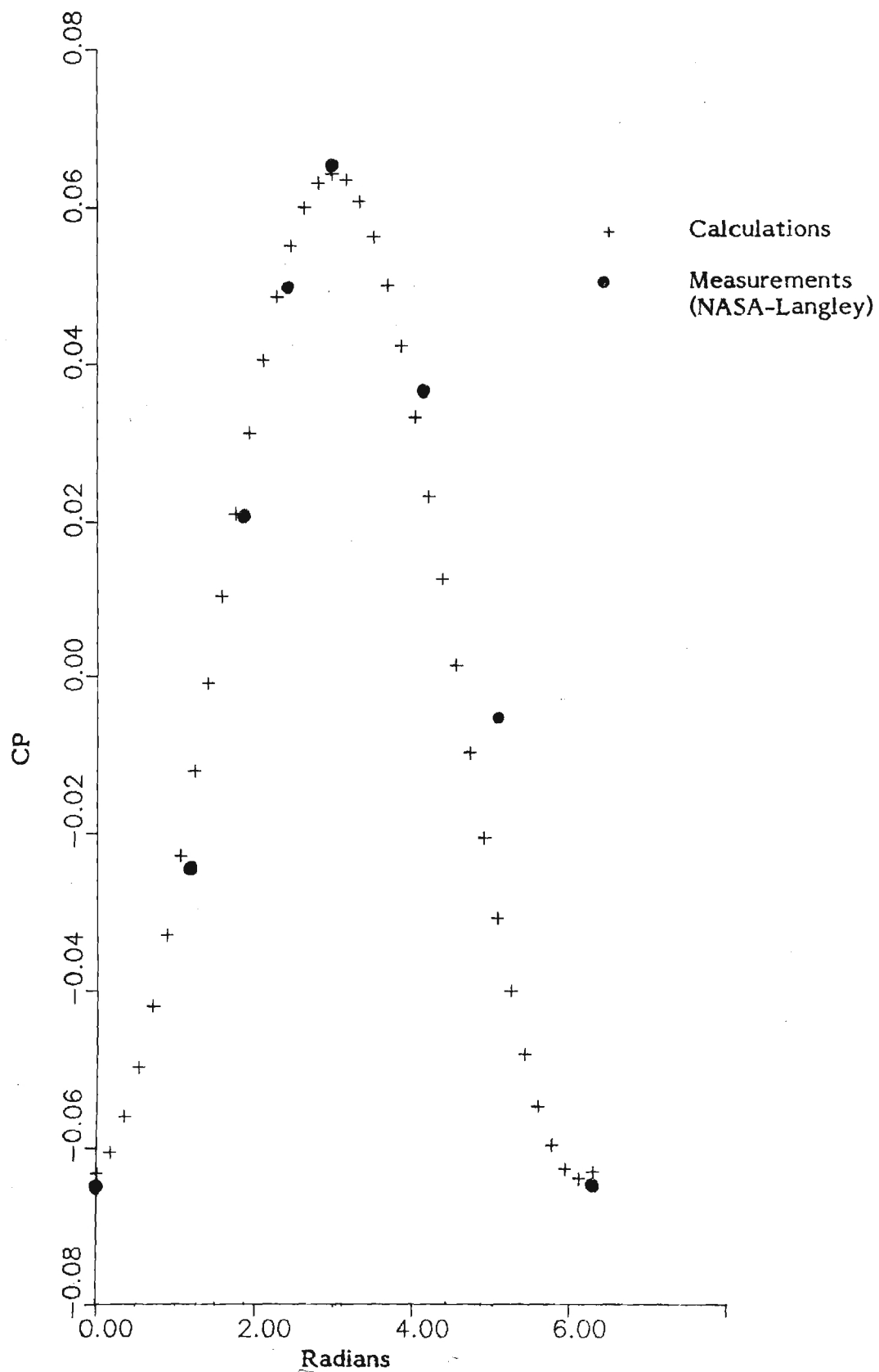


Figure 4. Comparison between calculations and measurements

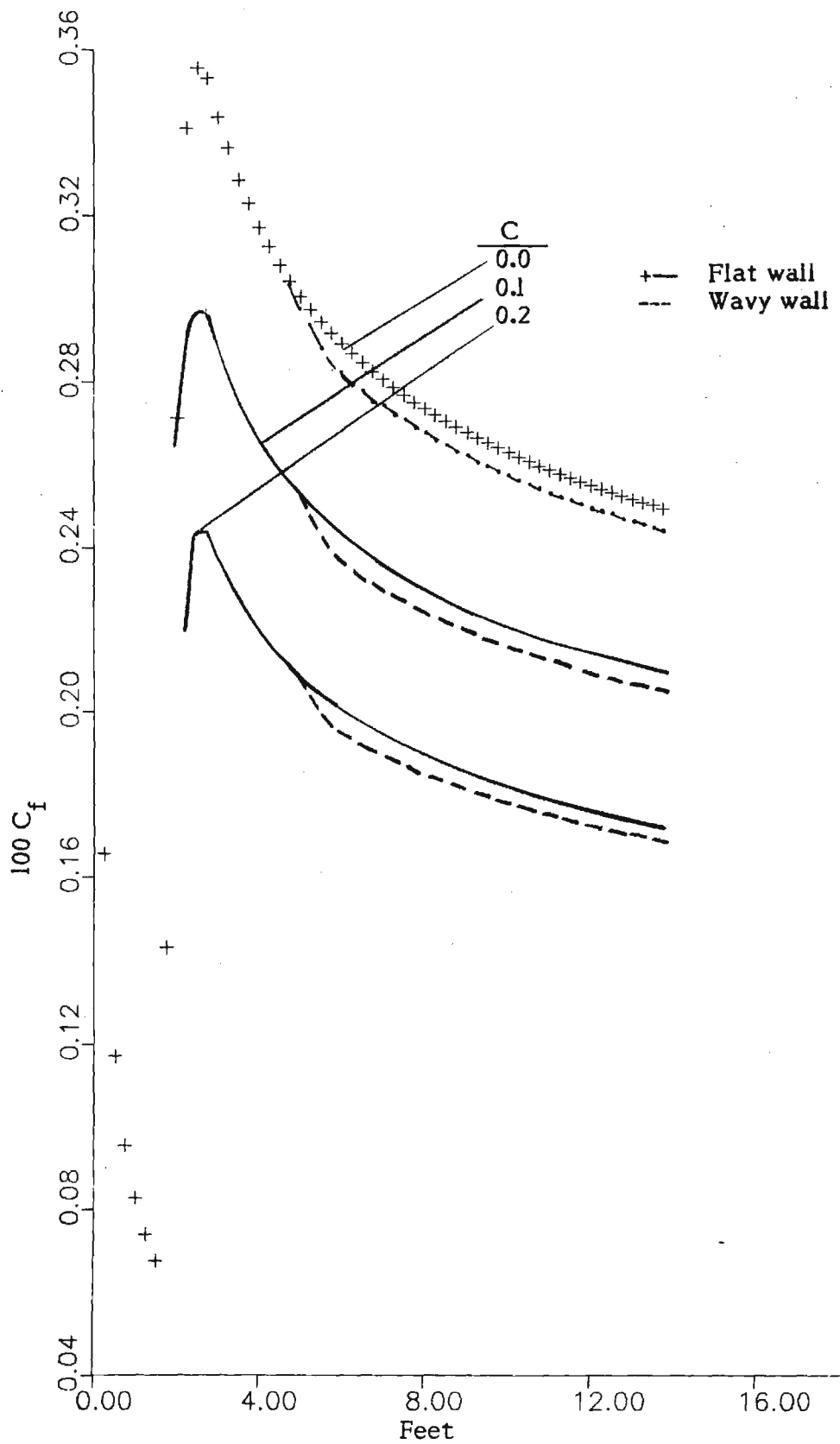


Figure 5. Mean skin friction predictions for the experimental set up NASA-Langley (Reference 4).

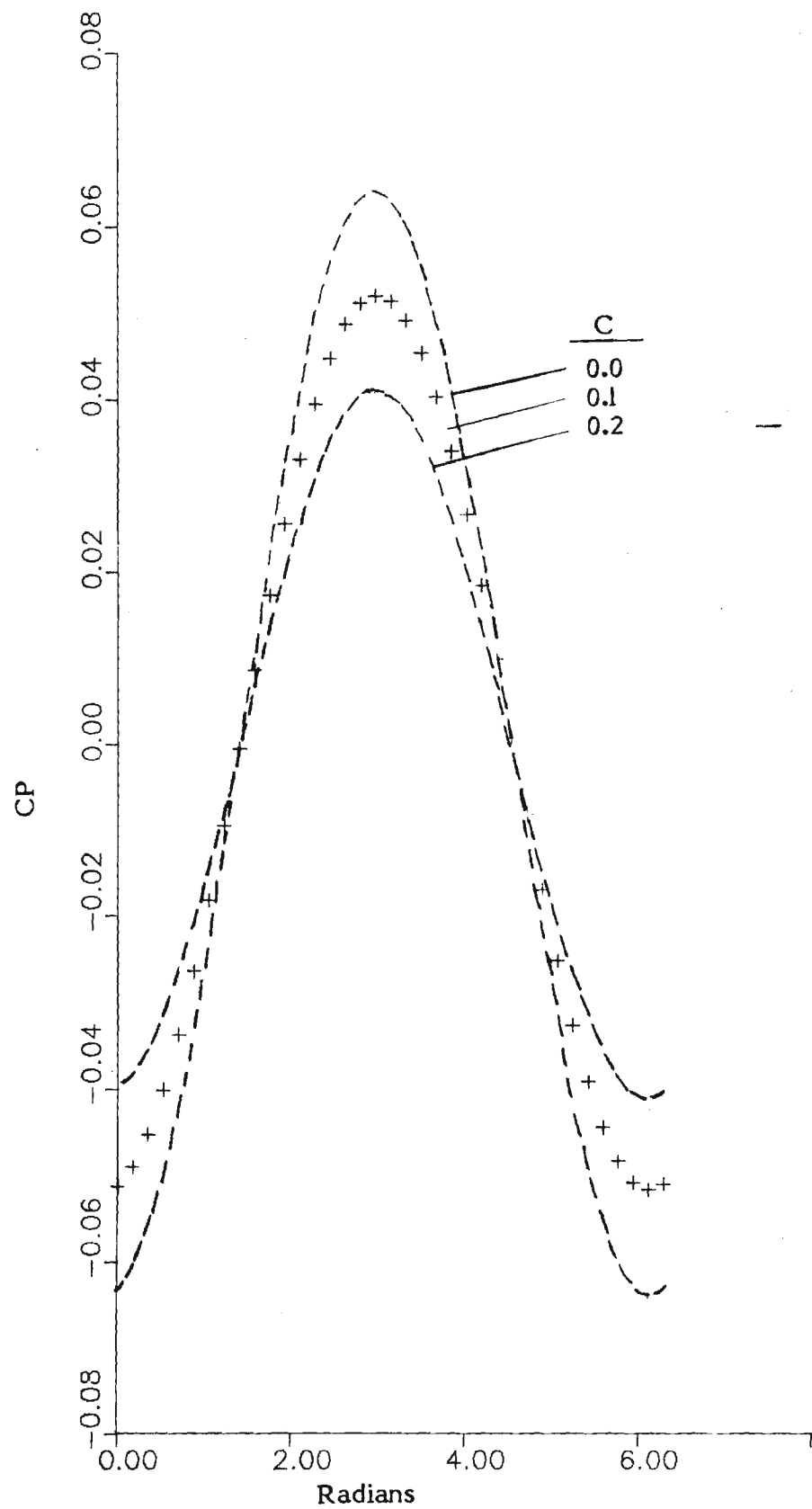
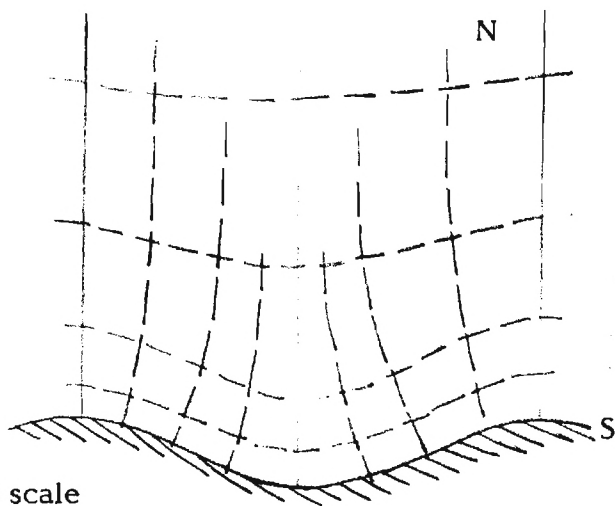


Figure 6. Predicted pressure distributions for the experimental set up of NASA-Langley (Reference 4)

wave velocity:  $c$   
 direction of wavy wall  
 motion:  $+x'$   
 direction of the wall  
 phase:  $+x$



(A - A) not to scale

$$\begin{aligned}
 S &= (x - c_1 t) + \epsilon e^{-ky} \cos k(x - ct) \\
 N &= y + \epsilon e^{-ky} \sin k(x - ct) \\
 Z' &= z - c_2 t
 \end{aligned}$$

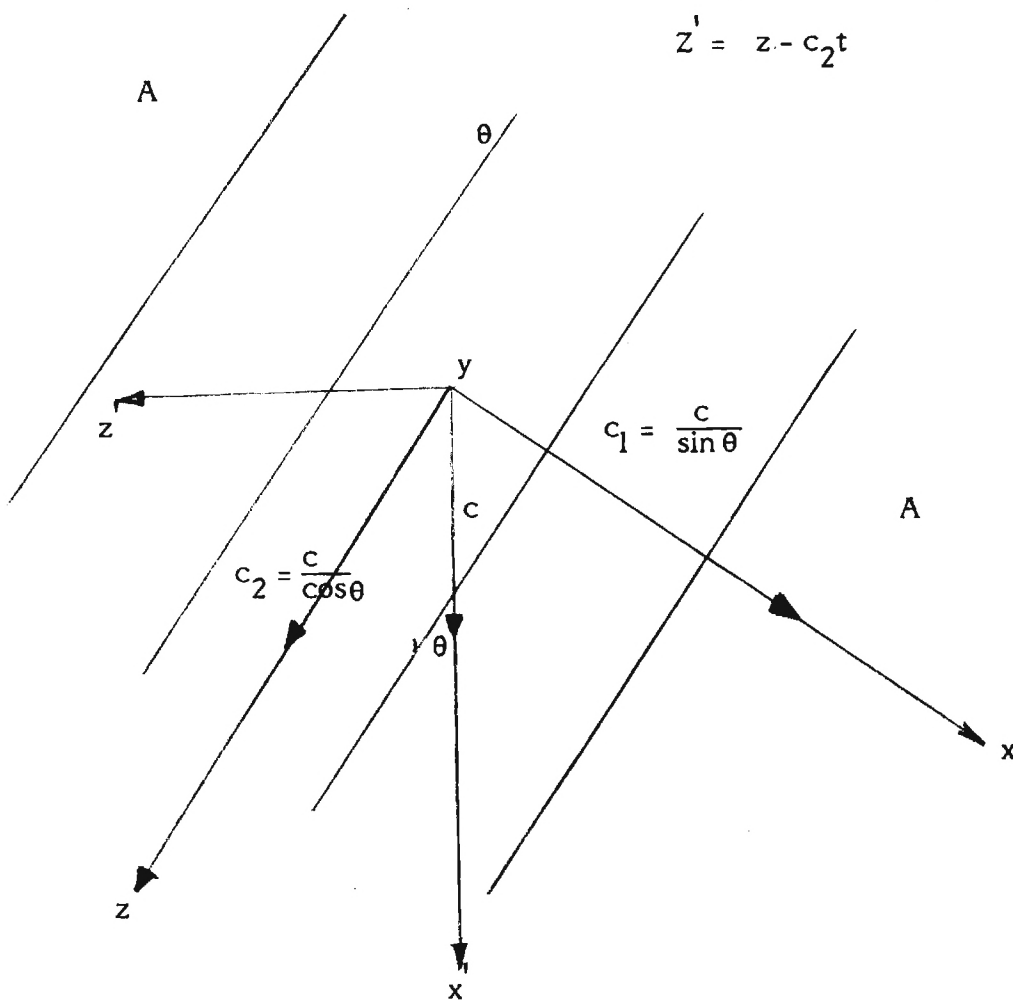


Figure 7. Coordinate system for the three-dimensional problem